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ESASGD 2016

GIS-IDEAS (2016)

Conference Title: International Conference on GeoInformatics for Spatial-Infrastructure Development in Earth & Allied Sciences (GIS-IDEAS)

FIRST-ORDER THEORY OF PERTURBED ORBIT CALCULATING

Luong Ngoc Dung[a] ,Nguyen Thai Chinh[b] , Tran Dinh Trong[a], Nguyen Dinh Huy[a] , Vu Dinh Chieu[a], Ha Thi Hang[a], Bui Ngoc Son[a]

[a]*Department of Geodesy, National University of Civil Engineering*

[b]*Department of Geodesy, Hanoi University of Mining and Geology; TU Berlin, GFZ German Reasearch Center For Geosciences*

Abstract

Geodesy is the Earth science that studies the form of the Earth (including its geometry and its gravity), its rotation and orientation in space. Thanks to the development of space techniques, the study of crustal deformations by using GNSS (Global Navigation Satellite System), the ocean topography by satellite altimetry, the temporal variations of the gravity field (mass transports), as well as the construction and monitoring of the International Terrestrial Reference Frame (ITRF) are some examples of the contribution of these techniques to the Earth observation including the current global change. Determining the gravity field of the Earth by satellite orbital analysis (to be monitored by the laser measuring system, DORIS or GNSS), as well as research axis and Earth's polar motion (by interferometry system, Very Long Baseline Interferometry) has indicated that POD (Precise Orbit Determination) is a strict requirement. The aim of this study is to introduce equations of movement, geometry problems and coordinates transformation of the orbit calculating. According these basis, building a formula based on quasi-circular analytical theory of orbit perturbed first-order due to the Earth's gravity field. In practical, tests are effected by quasi-circular and Kaula theories and the comparison of the results with a numerical integration based upon Eigen type models is convincing.

Keywords: Precise Orbit Determination - POD; perturbed orbit theory; Earth’s gravity field.

Introduction

Consider the relation between a central body of volume M (assuming the Earth) and another body of volume m (assuming an artificial satellite in an orbit around the Earth), we can establish the equations of non-perturbe orbital movement from the formula of two body problem.

According to the Newton’s second law, the trajectory of second body is like a conic, where its foci is the gravity center of central body. The motion of satellite around the central body are described by three Kepler’s laws:

- The planet (assuming the satellite) describes the elliptical trajectories, where the Sun (assuming the Earth) is one of two foci.
- Area law : the rayons vectors of planets sweep equal area during the same time.
- Square of the revolution period (T) of one planet is directly proportional to the cube of semi-major (a) of the planet’s elliptical trajectory: $n^2\ a^3 = GM = \mu$, where n = 2p/T and G is a constant of universal gravitation. The state vector of satellite is configured by its position and velocity in coordinates or the orbital elements.

1. Orbital dynamics

At first, the orbital dynamic has the object to calculate the ephemeris of bodies in the space. The essential elements of orbital dynamics are based on the constructions of a model with the forces present, the choice of reference frame and the integration of movement equations.

The types of trajectories can be much varied, but our main work focus on quasi-circular orbits around the Earth. We calculate trajectories either without taking into account the observations, that is extrapolation of initial conditions, or by taking into account the observations, that is the restitution (or compensation) of parameters of initial conditions.

That is therefore indispensable, in the orbit calculating, to approached the questions: 1) the choice of space frames, the time to write and integrate the movement equations on the one hand and to model the observations on the other hand. 2) the connections between different frames. 3) the parameterisation of movement - in Cartesian coordinates, classical keplerian elements or canonical variables, etc-.

The representation of orbit perturbation is mainly based on analytical theory in keplerian orbital elements (Kaula, 1966). However, the nature of space missions impose often to know the expression of perturbations in term of coordinates. The benefit of using directly coordinates is twofold in the development an analytical theory of satellite and for the applications in space geodesy. We are interested in analytical theory because it allows us to study soon the general or global characteristics of trajectories. Here are some basic criterion related to the establishment of analytical solution of integration of movement equations: 1) the theory is unique, and the relations are the same for all studying cases. 2) the literatures relations between causes (model coefficients) and effects (pertubative terms on the trajectories) are written only one time. We don't manipulate any data and adjustment of initial conditions is carried out in such an empirical way to find, for every period, the set of initial mean parameters of orbit.

1.1. The fundamental equations

Equations of movement: To spot the satellite in the space, we use either position $\vec{x}$(x,y,z) and velocity $\dot{\vec{x}}\ (\dot{x}, \dot{y}, \dot{z})$ vectors or orbital elements. In a fix and inertial frame, the Lagrangian expression can be written as:

$$L = T + U = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{\mu}{r} \qquad (1)$$

μ = GM: produce of gravitational constant - G and the mass of the Earth - M

$(\dot{x}, \dot{y}, \dot{z})$: the components of velocity vector and geocentrical rayon r

T : kinetic energy

U: potential (depends on the positions)

Dynamical equations, from fundamental principle, have an expression simple in geocentrical rectangular coordinates. Thus, it can be described in an inertial frame for a satellite of mass m :

$$\ddot{x} = -\frac{\mu}{r^3}x + \sum_i \frac{\partial U_i}{\partial \vec{x}} + \sum_j \frac{F_j}{m} \qquad (2)$$

(x, y, z) the components of position vector $\vec{x}$

U_i : the perturbed potentials (potential of fix gravity, variable parts, potential of tides, etc.)

F_j : the perturbed forces

The mechanism integration of these equations is not simple if the general solution is taken into account. In case a perturbed potential appears, the system of movement equations is no longer integrable by analytical method.

These equations are practical for numerical integration. In contrast, by analytical method, if we want to get round the particular difficulty to the integration, we can consider unlimited development of solution "around" one simple geometrical solution, that is the problem of two bodies. In these conditions, the choice of parameters (coordinates or elements) and the choice of integration frame are significant. But changing of parameters, the expression of perturbed gravity potential is expressed *"a priori"* in spherical coordinates in a frame related to the center mass of the Earth, need to be transformed.

1.2. Kaula theory in orbital elements

We present here a short reminder of Kaula theory, which uses classical keplerian orbital elements - *E ={a, e, i, Ω, ω, M}*, in the framework of lagrangian approach.

$$\begin{cases} \dfrac{da}{dt} = \dfrac{2}{na}\dfrac{\partial U}{\partial M} \\ \dfrac{de}{dt} = -\dfrac{\sqrt{1-e^2}}{na^2e}\dfrac{\partial U}{\partial \omega} + \dfrac{1-e^2}{na^2e}\dfrac{\partial U}{\partial M} \\ \dfrac{di}{dt} = -\dfrac{1}{na^2\sqrt{1-e^2}\sin i}\dfrac{\partial U}{\partial \Omega} + \dfrac{\cos i}{na^2\sqrt{1-e^2}\sin i}\dfrac{\partial U}{\partial \omega} \\ \dfrac{d\Omega}{dt} = \dfrac{1}{na^2\sqrt{1-e^2}\sin i}\dfrac{\partial U}{\partial i} \\ \dfrac{d\omega}{dt} = \dfrac{\sqrt{1-e^2}}{na^2e}\dfrac{\partial U}{\partial e} - \dfrac{\cos i}{na^2\sqrt{1-e^2}\sin i}\dfrac{\partial U}{\partial i} \\ \dfrac{dM}{dt} = n - \dfrac{2}{na}\dfrac{\partial U}{\partial a} - \dfrac{1-e^2}{na^2e}\dfrac{\partial U}{\partial e} \end{cases} \qquad (3)$$

In this case, the elements suffer from the variations of coefficents over time. On the other way, we show that it is not only the divisions by zero to *Ω* and *ω* (if *i = 0*) or *ω*, *M* (if *e = 0*), but also two elements *Ω*, *ω* are not defined in these cases. Then if we replace by the sets of six variables "non singular", the equations are more complicated, because of the mixing of metrical and angular elements.

1.3. Quasi-circular theory in spherical coordinates

Considering the "natural" development of terrestrial gravity potential in spherical coordinates, as soon as considering circular or quasi-circular trajectories, we choose the parameters as spherical coordinates (r, φ, λ). They allow us to represent in a simple way (in case non-perturbed) almost the cases envisaged. This avoids the disadvantage of Lagrange classical equations, that is Kaula theory (in orbital elements) in the case of eccentricity nil (*e=0*).

Theory of (Bois, 1992), as based theory on equations of second degree by time, allows to take into account of potentials and forces. If we choose a geocentrical frame to represent the trajectory of satellite, we have a relation between the coordinates (x, y, z) and (r, φ, λ) in equatorial plane by eq.4.

$$\begin{cases} x = r\cos\varphi\cos\lambda \\ y = r\cos\varphi\sin\lambda \\ z = r\sin\varphi \end{cases} \qquad (4)$$

But if we consider in the orbital plane of the trajectory, the geometrical solution becomes ($r = r_0$, $\varphi=\varphi_0$, $\lambda= n(t-t_0)+\lambda_0$). They can serve from base (solution called zero order) to develop a perturbed solution.

Then this implies to establish the movement equations of satellite, in using spherical coordinates in the orbital plane, in relation to the inertial fix frame. Therefore it has to calcul the composition of velocity (derivative of position) between fix frame and our frame of integration. In this case, there are two rotations to carry out : one arounds Oz of $\Omega(t)$ (angle corresponding to node of trajectory and its temporal derived due to flattening of the Earth) and one arounds Ox of i (inclination of orbital plane with respect to fix plane Oxy that we consider as constant).

Following the Lagrange algorithm, we form the movement equations by spherical coordinates in fix frame Oxyz. In the kinetic energy eq. 5

$$T = \frac{1}{2}\left(\dot{r}^2 + (r\dot{\varphi})^2 + \left(rcos\varphi\dot{\lambda}\right)^2\right) \qquad (5)$$

$(\dot{r})$, $(r\dot{\varphi})$, $(rcos\varphi\dot{\lambda})$ are three components of coordinates of the velocity which give us the movement equations (6)

$$\begin{cases} \ddot{r} = r\cos^2\varphi\dot{\lambda}^2 - \frac{\mu}{r^2} \\ \ddot{\varphi} = -2\frac{\dot{r}}{r}\dot{\varphi} - cos\varphi sin\varphi\dot{\lambda}^2 \\ \ddot{\lambda} = -2\frac{\dot{r}}{r}\dot{\lambda} + \frac{2sin\varphi}{cos\varphi}\dot{\lambda}\dot{\varphi} \end{cases} \qquad (6)$$

Considering the movement equations in the mean precesse orbital plane (Ox'y'z') due to the rotations $\Omega(t)$ and i, we have new kinetic energy eq.7. The coordinates of satellite in this plane is described by the relation $(x, y, z)^T = R3(-\Omega(t))R1(i)$ $(x', y', z')^T$.

$$T = \frac{1}{2}\left(\dot{r'}^2 + (r'\,\dot{\varphi}' r'\dot{\Omega}\sin i\,cos\lambda')^2 (r'cos\varphi'\left(\dot{\lambda}' - \dot{\Omega}cosi\right) - r'sin\varphi'\,\dot{\Omega}sin\,i\,sin\lambda')^2\right) \qquad (7)$$

From eq.7, we rewrite the expression of movement eq.8 that is a function of parameters of coordinates (r', φ', λ') and rotation angles (Ω, i).

$$L = T + U(r', \varphi', \lambda'; i, \Omega) \qquad (8)$$

According to first order theory of (Exertier and Bonnefond, 1997) to separate spherical coordinates in two parts : 1. Initial solution (r'_0, φ'_0, λ'_0 : problem of two bodies) and 2. Perturbations on the orbit (r'_1, φ'_1, λ'_1). We then developed eq.8 to obtain new movement eq.9 of perturbed components

$$\begin{cases} \varepsilon\ddot{r}'_1 - 3\varepsilon n^2 r'_1 - 2\varepsilon r'_0 n\dot{\lambda}'_1 - 2\varepsilon r'_0 n\dot{\Omega}\cos i = -\frac{\partial U^*}{\partial r'_1} \\ \varepsilon\ddot{\varphi}'_1 + \varepsilon\varphi'_1(n - \dot{\Omega}\cos i)^2 + 2\varepsilon n\dot{\Omega}\sin i\sin\lambda' = \frac{1}{r'^2_0}\frac{\partial U^*}{\partial\varphi'_1} \\ \varepsilon\ddot{\lambda}'_1 + \frac{2}{r'_0}\varepsilon'\dot{r}'_1(n - \dot{\Omega}\cos i) = \frac{1}{r'^2_0}\frac{\partial U^*}{\partial\lambda'_1} \end{cases} \qquad (9)$$

2. Formula of perturbed terrestrial potential

The expression of gravitational perturbed potential expressed in spherical coordinates in a frame related to center of mass of the Earth would be as follow eq.10 (with r_0, φ_0, λ_0 in the fix frame of the Earth):

$$U^* = \frac{\mu}{r_0}\sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{a_e}{r_0}\right)^l P_{lm}(\sin\varphi_0)(C_{lm} - jS_{lm})\exp jm\lambda_0 \qquad (10)$$

a_e : equatorial radius (km)

C_{lm}, S_{lm} : Normalize spherical harmonic coefficients described the gravity potential.

P_{lm} : Associated functions of Legendre.

r_0, φ_0, λ_0 : Spherical coordinates of satellite.

Following the parameters used, we developed perturbed potential to the satellite, with the variables of the solution. By keplerian orbital elements (Kaula) or spherical coordinates in the orbital plane (in case quasi-circular orbit) we then evaluate the perturbations.

2.1. Form of perturbations in keplerian elements

We note here a model of perturbed potential by function of orbital elements of satellite (Kaula) in inertial frame (Zarrouati, 1987).

$$U^* = \frac{\mu}{a}\left(\frac{a_e}{a}\right)^l\sum_{p=0}^{l}F_{lmp}(i)\sum_{q=-\infty}^{+\infty}G_{lpq}(e)S_{lmpq}(\omega, M, \Omega, \theta) \qquad (11)$$

where

$$S_{lmpq} = \left(\begin{bmatrix} C_{lm} \\ -S_{lm} \end{bmatrix}_{(l-m)_{odd}}^{(l-m)_{even}} \cos\psi_{lmpq} + \begin{bmatrix} S_{lm} \\ C_{lm} \end{bmatrix}_{(l-m)_{odd}}^{(l-m)_{even}} \sin\psi_{lmpq}\right)$$

With the sidereal angle of the Earth:

$$\psi_{lmpq} = (l-2p+q)M + (l-2p)\omega + m(\Omega - \theta)$$

One important step in the transformation U^* (11) is necessary to take account the terms :

$$\frac{a_e^l}{r_0^{l+1}} = \frac{a_e^l}{a^{l+1}}\frac{a^{l+1}}{r_0^{l+1}} = \frac{1}{a}\left(\frac{a_e}{a}\right)^l\left(\frac{a}{r_0}\right)^{l+1}$$

where the semi-major axis "a" is introduced in equation 3.9 of (Kaula, 1966):

$$\frac{a}{r_0} = \frac{1 + e\cos v}{1 - e^2}$$

Eq.11 introduces two functions of inclination $F_{lmp}(i)$ and eccentricity $G_{lpq}(e)$, where $G_{lpq}(e)$ functions allows to transform, in the orbital plane, the formula in functions of polar coordinates r and υ that depend on keplerian elements e and M:

$$\left(\frac{a}{r_0}\right)^{l+1}\exp i.((l-2p)\upsilon) = \sum_{q=-\infty}^{+\infty}G_{lpq}(e)\exp i.((l-2p+q)M)$$

An integration of eq.11 allows to identify the signature of harmonic spherical coefficients (with the set *lmpq*) intervening in the expression of perturbed potential. So we developed tools to identify the amplitudes and periods of perturbations (in terms of short, average, long periods and secular effects of angles) by frequencys of *lmpq* sets: $\dot{\psi}_{lmpq} = (l - 2p + q)\dot{M} + (l - 2p)\dot{\omega} + m(\dot{\Omega} - \dot{\theta})$

So we see that the eq.11 is well adapted to the terms :

$$\begin{aligned} F'_{lmp}(i) &= \frac{dF_{lmp}}{di} \\ G'_{lpq}(e) &= \frac{dG_{lpq}}{de} \\ S'_{lmpq}(M, \omega, \Omega, \theta) &= \frac{dS_{lmpq}}{d\psi_{lmpq}} \end{aligned} \qquad (12)$$

That gives us the eq.13 which are always harmonical functions of ψ_{lmpq} argument defined in eq.11. The periodical terms are organized by set of index (*lmpq*) corresponding to frequency $\dot{\psi}_{lmpq}$.

$$\frac{\partial U_{lmpq}}{\partial a} = -\frac{l+1}{a}U_{lmpq}$$
$$\frac{\partial U_{lmpq}}{\partial e} = \frac{\mu}{a}\left(\frac{a_e}{a}\right)^l G'_{lpq}(e)\,\mathrm{F}_{lmp}(\mathrm{i})\,\mathrm{S}_{lmpq}(\mathrm{M},\omega,\Omega,\theta)$$
$$\frac{\partial U_{lmpq}}{\partial i} = \frac{\mu}{a}\left(\frac{a_e}{a}\right)^l G_{lpq}(e)\,\mathrm{F}'_{lmp}(\mathrm{i})\,\mathrm{S}_{lmpq}(\mathrm{M},\omega,\Omega,\theta) \qquad (13)$$
$$\frac{\partial U_{lmpq}}{\partial \Omega} = m\frac{\mu}{a}\left(\frac{a_e}{a}\right)^l G_{lpq}(e)\,\mathrm{F}_{lmp}(\mathrm{i})\,\mathrm{S}'_{lmpq}(\mathrm{M},\omega,\Omega,\theta)$$
$$\frac{\partial U_{lmpq}}{\partial \omega} = (l-2p)\frac{\mu}{a}\left(\frac{a_e}{a}\right)^l G_{lpq}(e)\,\mathrm{F}_{lmp}(\mathrm{i})\,\mathrm{S}'_{lmpq}(\mathrm{M},\omega,\Omega,\theta)$$
$$\frac{\partial U_{lmpq}}{\partial M} = (l-2p+q)\frac{\mu}{a}\left(\frac{a_e}{a}\right)^l G_{lpq}(e)\,\mathrm{F}_{lmp}(\mathrm{i})\,\mathrm{S}'_{lmpq}(\mathrm{M},\omega,\Omega,\theta)$$

2.2. Form of perturbations in spherical coordinates

We applied, for quasi-circula orbit, a form of perturbed potential by (Balmino, 1996). In this case, we are considering spherical coordinates of satellite in the orbital plane to evaluate directly the effects of perturbed potential U^* (eq.14) to orbit.

$$U^* = \frac{\mu}{r'_0}\left(\frac{a_e}{r'_0}\right)^l \sum_{k=-l}^{+l}\sum_{m=0}^{l} P_{lk}(\sin\varphi'_0)\,\mathrm{d}_{lmk}(\mathrm{i})(C_{lm} - jS_{lm})\exp j\psi_{km}$$
$$\psi_{km} = k\left(\lambda'_0 + \frac{\pi}{2}\right) + m\left(\Omega - \theta - \frac{\pi}{2}\right) \qquad (14)$$

According to the properties useful of Legendre polynomial and functions, we have derived expressions of perturbed potential in spherical coordinates corresponding to eq.15 :

$$\frac{\partial U^*}{\partial r'} = -\frac{\mu}{r'^2_0}\left(\frac{a_e}{r'_0}\right)^l (l+1)\sum_{k=-l}^{+l}\sum_{m=0}^{l} P_{lk}(\sin\varphi'_0)\,d_{lmk}(i)\left\{C_{lm}\cos\psi_{km} + S_{lm}\sin\psi_{km}\right\}$$
$$\frac{1}{r'^2_0}\frac{\partial U^*}{\partial \varphi'} = -\frac{\mu}{r'^3_0}\left(\frac{a_e}{r'_0}\right)^l \sum_{k=-l}^{+l}\sum_{m=0}^{l} P'_{lk}(\sin\varphi'_0)\,d_{lmk}(i)\left\{C_{lm}\cos\psi_{km} + S_{lm}\sin\psi_{km}\right\}$$
$$\frac{1}{r'^2_0}\frac{\partial U^*}{\partial \lambda'} = \dot{\psi}\frac{\mu}{r'^3_0}\left(\frac{a_e}{r'_0}\right)^l \sum_{k=-l}^{+l}\sum_{m=0}^{l} P_{lk}(\sin\varphi'_0)\,d_{lmk}(i)\left\{C_{lm}\cos\psi_{km} + S_{lm}\sin\psi_{km}\right\}$$

(15)

Obviously, we have three pairs of correspondence between (9) and (15) equations which help us to determine the constant, secular and periodical terms of perturbations in first-order by algorithm of integration (Bois, 1992). Difference from Kaula theory, we note that in this case the theory is developed with only sets of index (lmk) and the frequency $\dot{\psi}_{km} = k\dot{\lambda}'_0 + m(\dot{\Omega} - \dot{\theta})$, with $\dot{\lambda}'_0 = n$

3. Calculating of applications

3.1 Inclination functions

We have discussed above two approaches to calculate perturbations of first-order: 1, Kaula theory with orbital elements for satellite trajectories different zero and 2, Analytical theory of (Bois 1992) then developed by (Exertier and Bonnefond, 1997) with spherical coordinates for quasi-circular trajectories. Analyzing of two theories shows that, for Kaula approach, it is difficult to determine elements in case of orbital plane inclination (i) or factor eccentricity (e) are zero. On the other hand, calculating of partial derived in spherical coordinates is not simple, because of the difficulty when calculating inclination function and Legendre polynomial and functions. The values of Legendre functions increase very quickly according to l index growth, it is not favorable for numerical calculating. In addition, the efficiency and stability of the method to calculate inclination functions are challenging

In our case, we developed tools to calculate inclination functions $\bar{d}_{lmk}(i)$ in normalized harmonic coefficients by stable recurrences of (Sneeuw, 1992). The table of Fig.1 shows the structure of harmonic coefficients that has a dimension of index $l = 2 \div l_{Max}$, $m = 0 \div l$, $k = -l \div l$

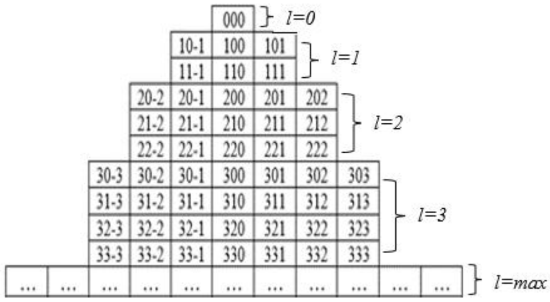

Fig.1 Calculating diagram of harmonic coefficients

After both of tests for these recurrences of Sneeuw with normalized (in red) and denormalized (in black) coefficients by then in comparison with calculation by polynomials and functions of Legendre (in green). The results Fig.2 shows that the denormalized version is only stable up to degree 85 while normalized version can be stable more than index 300 (this case shows only up to degree 150).

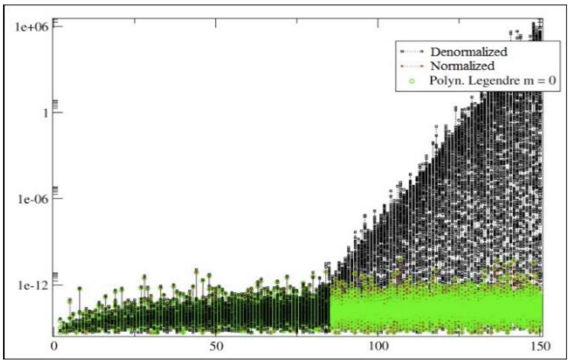

Fig.2 Stability of recurrences in normalized (red) and denormalized (black) versions

3.2 Spectrum of perturbations

Thanks to inclination functions $\bar{d}_{lmk}(i)$ with normalized harmonic coefficients, we calculated spectrum of perturbation from gravity field Eigen models, which are built by harmonic coefficients C_{lm}, S_{lm}, in some different orbits.

The construction of coefficients set up to degree and order 50 in gravity field model EIGEN-GRGS.RL02bis.MF. This allows us to calculate frequencies of perturbations acting on the orbits of Jason-2, Saral and LAGEOS1&2 (Fig.3). In our calculation for 10 days, we neglected terms under 0.01m and the spectrums show us radial amplitude (in meter) of coefficients of sets (lmk).

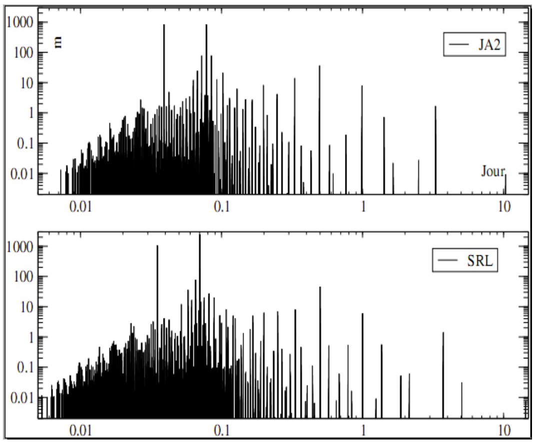

Fig.3a Periods of radial perturbations (in meter) from coefficients of EIGEN-GRGS.RL02bis.MF gravity field model for: Jason-2 and Saral

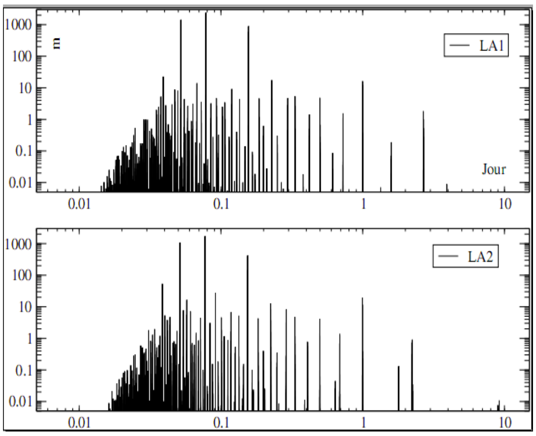

Fig.3b Periods of radial perturbations (in meter) from coefficients of EIGEN-GRGS.RL02bis.MF gravity field model for: LAGEOS-1&2

Although there is no difference in the forms of spectrums, but we can see the most density of sets in case of Saral. This shows the impact of perturbation cause of gravity potential depending on altitude and inclination factors.

3.3 Discussion of results

Evaluating ability and stability of analytical quasi-circular (in case e < 0.003) and Kaula (in case e > 0.003) theories.

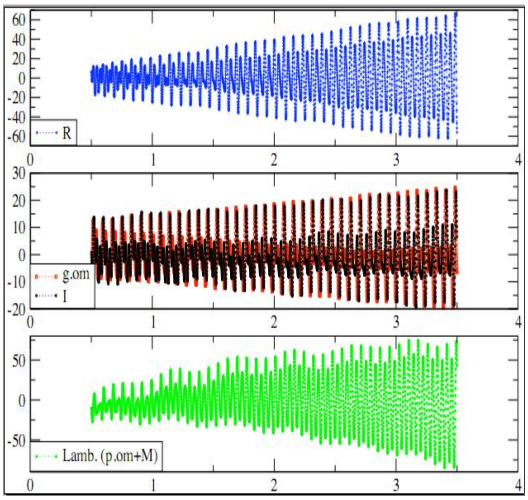

Fig.4a Differences (in meter) between quasi-circular theory and numerical integration for satellite orbit: Jason-2

Tests are effected to determine amplitude of perturbed sets (coefficients of each sets *lmk* for quasi-circular case and *lmpq* for Kaula case, respectively) of EIGEN-GRGS.RL02bis.MF gravity potential.

We chose satellite orbits of Jason-2 (at altitude 1335km and inclination 66°), Saral (altitude 780 km, inclination 98°) to test quasi-circular theory and two satellite orbits LAGEOS-1&2 (at about 6000km of altitude, 110° of inclination) to test Kaula theory. All of the satellite orbits are extrapolated by using static gravity model EIGEN-GRGS.RL02bis.MF (GDR-D generation based on 8 years of GRACE+LAGEOS data).

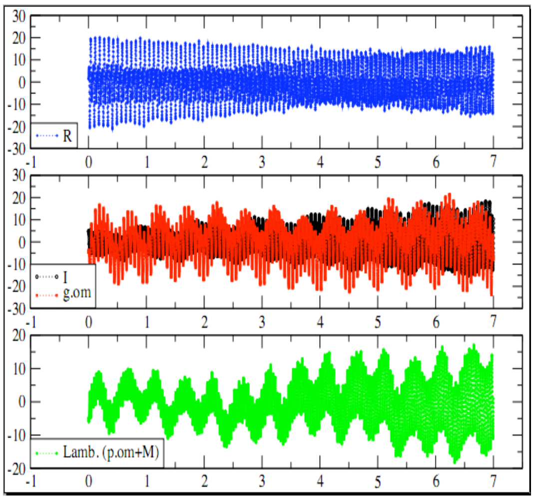

Fig.4b Differences (in meter) between quasi-circular theory and numerical integration for satellite orbit: Saral

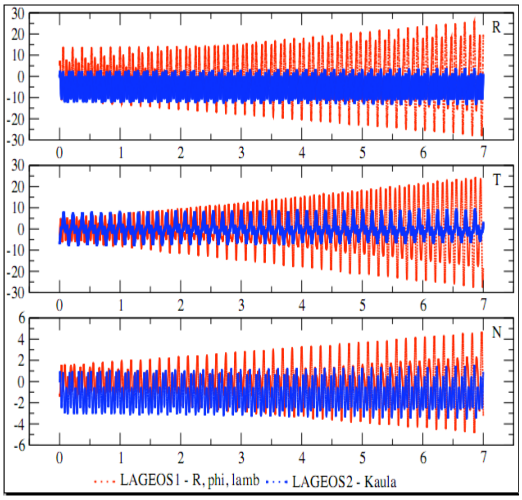

Fig. 4c Differences (in meter) between Kaula theory and numerical integration for satellite orbit: LAGEOS1&2

The results of some comparisons, between analytical and numerical theories in case of Kaula theory for LAGEOS1&2 (Fig.4c) and quasi-circular theory for Jason-2 (Fig.4a) and Saral (Fig.4b), show that differences in the order of magnitude of a coupe of tens of meters on a, e, i or r elements and around ten of meters on the angles. Note that we have no initialization process of mean elements at the beginning, that maybe contribute some improvement. In addition it is important to take into account of secular effects of second order $C_{2,0}^{2}$ on the angles. This problem would be a part of the future work.

4. Conclusion

In this article, we introduced equations of movement in orbit calculating. Beside to show the Kaula theory in orbital elements, we developed an analytical theory in spherical coordinates (call quasi-circular orbit) for orbits with very small eccentricity ($e < 0.003$). These analytical theories will allow us to calculate almost the cases of orbital trajectory. In addition, they are more advantage than dynamical methods to predict the general characteristics of trajectories. In pratically, we are based on analytical theories to calculate amplitude of perturbations of EIGEN-GRGS.RL02bis.MF gravity field model in order several kilometres (Fig.3a & 3b), which gave us a relative precision of 10^{-3} to order of tens meters in orbit calculating (Fig.4a,b,c). It is important to continue to study this consequence in case of error propagation (by analytical integration) less than 1m to detect the quantities less than 1 mm in calculating orbit.

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